6.02 Introduction to Algebra

What is algebra?

Algebra is a system for working with general quantities taking on different values, known as variables. These quantities are often denoted by single letters, e.g. x.

- Topics in algebra include:
- Simplifying expressions
- Solving equations involving one variable
- Factorising expressions
- Problems involving powers of variables
- Problems involving algebraic fractions

Why should I learn algebra?

If you need to study algebra as part of your degree course, it is probably because algebra is an important skill for your course profession.

Apart from this, some people enjoy learning to do algebra because it provides a language for describing regular structures and processes, such as patterns.

Variables

The most important idea in algebra is being fluent with thinking of a **variable** as a **general quantity which can take on any value within a range of values**. This range of possible values can be restricted to whole numbers or it can be any point on the number line (real numbers). It may be bounded (finite) or unbounded (infinite).

Example: Let T be a temperature in degrees Celsius. T can take on any value and has no upper limit but it has a lower limit of absolute zero, which is -273.15.

Terms

A **term** is a product of a constant with one or more variables which may be raised to powers. Terms are written without any multiplication or division signs.

Example: A term corresponding to the kinetic energy of a body is $\frac{1}{2}mv^2$, where m is its mass and v is its velocity. $\frac{1}{2}mv^2$ means one half of m multiplied by the square of v.

Expressions

An **expression** is a more general combination of constants and variables than a term. One simple combination is to add or subtract terms.

Example: Suppose a square has sides of length $x + y$. Then the area of the square is $x^2 + 2xy + y^2$. Both $x + y$ and $x^2 + 2xy + y^2$ are examples of expressions.

Functions

Functions are a shorthand way of writing expressions. Functions are represented by letters just like variables. However functions depend on one or more variables which are normally given in brackets after their name.

Functions are defined using an **equals sign** which assigns the function and the expression with which it is associated to have the **same numerical value**.

Example: Consider a circle with radius given by the variable r. The area of this circle is thus πr^2. We can then define a function A which depends on the variable r to represent its area by the equation $A(r) = \pi r^2$.

Equations in algebra

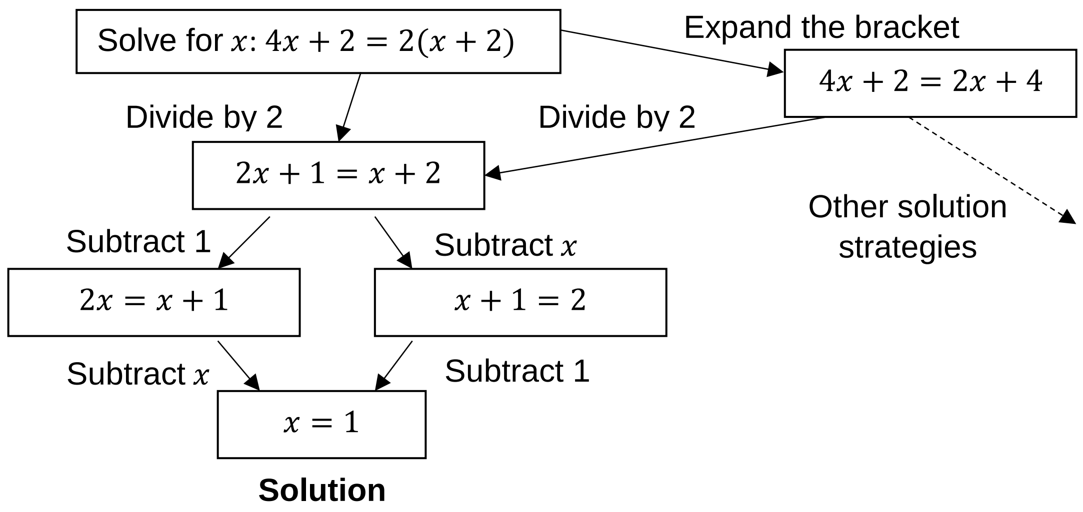
Equations are used in different ways in algebra. They can:

1. Define the value of a variable, e.g. $y = 5$
2. Define a function, e.g. $f(x) = x^2 + 3x + 1$
3. Express a constraint between one or more variables that is **always true**, e.g. $x + y = 3$

Equations are used in the process of finding out the value of unknown variables. This is called **solving equations**. A formula is a restricted type of equation. It is of type 3 in the list above.

Solving equations

When solving an equation or a system of equations, there are often choices to be made. Some choice will be better than others. The best choice can only really be learnt through experience. This is illustrated by the diagram below:



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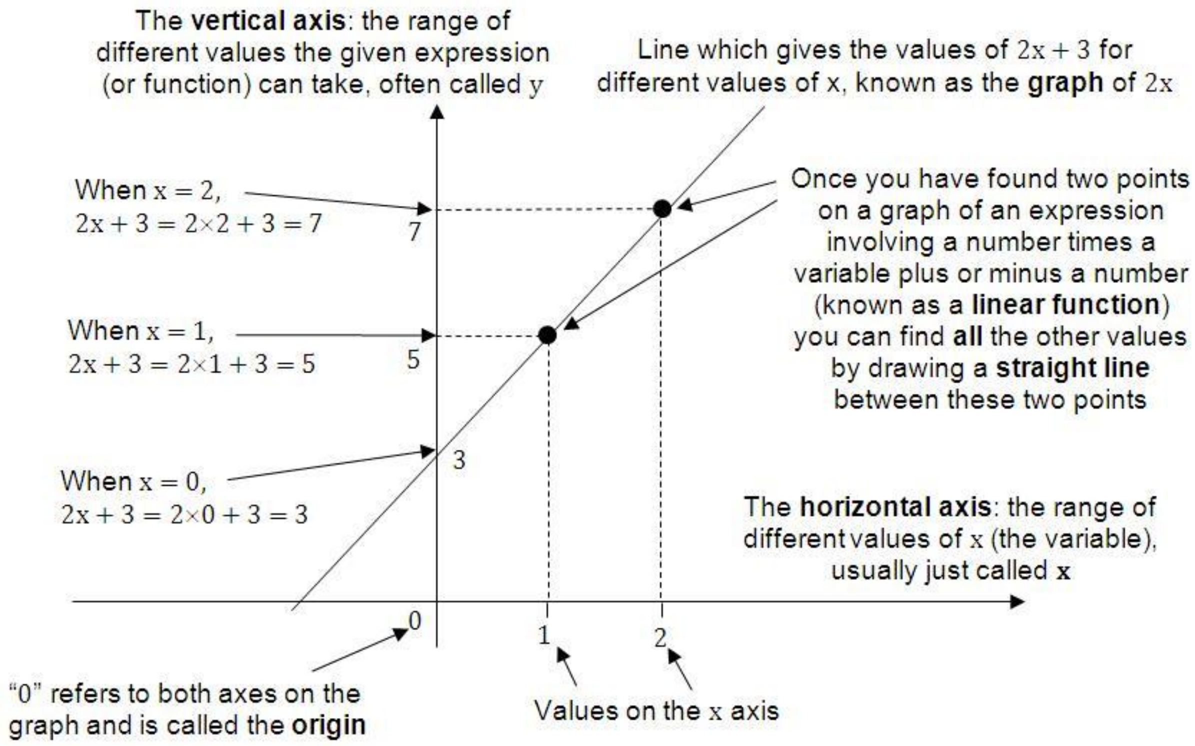
Graphs

A graph displays the relationship between two variables, or the relationship between a function and its dependent variable. The simplest type of graph is that for a **linear function**.

Below is a graph of the equation $y = 2x + 3$:



Suggested further reading

Mason, J., Graham, A. & Johnston-Wilder, S. (2005) *Developing Thinking in Algebra*, London: Paul Chapman Publishing.